

# DAY 8

# Profitable Pictures

Students begin to examine the relationship between the profit function and the feasible region.

## Mathematical Topics

- Expressing constraints symbolically
- Finding a feasible region
- Finding the value of a profit function for some points in a feasible region
- Exploring the set of points that yield a given profit
- Trying to maximize a profit function

## Outline of the Day

### In Class

1. Discuss *Homework 7: Picturing Pictures*
  - Post a graph of the feasible region for later use
  - Identify the profit function
2. *Profitable Pictures*
  - Students find combinations that yield a given profit
  - The activity will be discussed on Day 9

### At Home

*Homework 8: Curtis and Hassan Make Choices*

### Special Materials Needed

- Transparency of the feasible region for *Homework 7: Picturing Pictures* (see Appendix B)

## 1. Discussion of *Homework 7: Picturing Pictures*

As a class, decide which variable will go on which axis. We will place  $p$  on the horizontal axis and  $w$  on the vertical axis, but you should inform students that either choice is correct. Ask the class how to represent the

two constraints symbolically, perhaps choosing students at random. They should get inequalities equivalent to these:

$$p + w \leq 16 \quad (\text{for the number of pictures})$$

$$5p + 15w \leq 180 \quad (\text{for the money available for materials})$$

Students may remember to include the inequalities  $p \geq 0$  and  $w \geq 0$ . If not, you can bring this up as they sketch the feasible region, as described below.

• **Question 2: The feasible region**

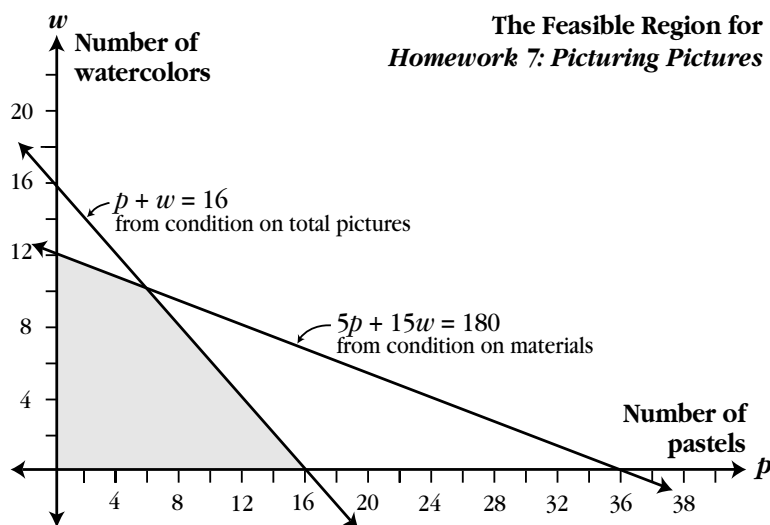


Ask for a volunteer to present the development of the feasible region, and post the graph for further use today and tomorrow.

If your students have a shaky grasp of the connection between inequalities and their graphs, you should use this as another occasion to review the process. Have students offer specific points that fit the constraints, and then have the class check that they

satisfy all of the constraints (including  $p \geq 0$  and  $w \geq 0$ ). Continue plotting points until the connection between the developing graph and the equations  $p + w = 16$  and  $5p + 15w = 180$  becomes clear.

The feasible region for *Homework 7: Picturing Pictures* should look like the shaded area of the graph shown here. You will probably want a transparency of this region (in addition to the posted graph) for tomorrow's discussion of the next activity, *Profitable Pictures*. A large diagram for making this transparency is included in Appendix B.



**“What does the graph tell you?”**

Ask students to articulate what this graph means. They should recognize that each point in the feasible region (or at least each whole-number point) represents a possible choice Hassan can make about how many pictures of each type to make.

*Comment:* The issue of limiting the variables to whole numbers may come up again here, as it did for the cookie problem. That is, students may point out that they shouldn't be shading the entire region bounded by

the lines, but only marking the points with whole-numbers coordinates. If the issue is raised, you can review the ideas from the earlier discussion of this issue (see the subsection "Whole numbers only?" on Day 6).

### • Questions 3 and 4: The profit function

You can have several students each give the profit for one of the points they used in Question 3. You might put this information in an In-Out table or other type of chart as it is presented. Then ask for the profit function in terms of the variables  $p$  and  $w$ .

*Note:* Some students may have read the amounts \$40 and \$100 as selling prices rather than profits, because that was the way information was

presented in the unit problem. It is important that this issue be clarified before students begin work on today's activity, *Profitable Pictures*.

You may want to look at the various points that students suggested and the profits for these points to determine the maximum profit achieved so far. Tell students that they will learn a way to find the maximum profit and to *prove* that they have the maximum.

## 2. Profitable Pictures

Have students work in their groups on the activity *Profitable Pictures*. They will continue their work on this activity tomorrow. During tomorrow's work, each group will prepare a report on this activity. The report should include answers to Questions 1 through 4 but should be focused especially on Question 5.

*Note:* If groups have trouble finding combinations that yield a specific profit, you might suggest that they consider points both outside the feasible region and in the region.

## Homework 8: Curtis and Hassan Make Choices

This assignment is intended to strengthen students' understanding that profit (or cost) lines form a parallel family. No specific discussion is scheduled for this assignment, but

you may want to use students' work on these problems in conjunction with tomorrow's discussion of *Profitable Pictures*